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Existence and asymptotic stability of solutions of differential equations with delay

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We discuss existence and asymptotic stability of solutions of the parametric differential equation arising from population dynamics models

$$\frac{\partial u}{\partial t}(t,x) = -b(t,x)u(t,x) + g\left(t, u(t,x), \int_{-\tau}^{0} u(t+\theta,x)d\theta\right),\ t \geq t_0,\ x \in [0,1]. \qquad (1)$$

The term $\int_{-\tau}^{0} u(t+\theta,x)d\theta$ means that at every t the system has memory of the evolution of the state up to that moment for a past of fixed amplitude $\tau > 0$.

The study is carried out in the context of semilinear differential equations in Banach spaces. We see equation (1) as a particular case of the semilinear differential equation with functional delay

$$y'(t) = A(t)y(t) + f\left(t, y(t), y_t\right),\ t \geq t_0, \qquad (2)$$

where y_t stands for the function $y_t(\theta) = y(t+\theta)$, $\theta \in [-\tau, 0]$, $t \geq t_0$. The results are achieved on (2) by combining iterative methods and fixed point theorems for condensing maps, and then made to fall back on (1).

We also show results when systems driven by (1) are subject to impulsive forces $\mathcal{I}_k$ at times t_k, $\{t_k\}_{k\in\mathbb{N}}$ increasing diverging sequence, or to feedback controls $\omega(t,x) \in W(u(t,x))$, W set-valued map. Once these conditions are reread in the abstract setting, in the first case we get to work with functions I_k, $k \in \mathbb{N}$, defined in a suitable functions space. In the second, however, we are naturally faced with the semilinear differential inclusion $y'(t) \in A(t)y(t) + F\left(t, y(t), y_t\right)$, which requires the use of multivalued analysis tools.